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Their Reciprocal Allspace-Filling Transformations

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New Models of Synergetics Topology and Their Reciprocal Allspace-Filling Transformations

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Key words : foldability, axis of spin, periodicity, allspace-filling

All the unstable Archimedean polyhedral systems can be symmetrically transformed into Platonic polyhedral systems by accumulating edges and vertices of the respective same number at each of their normal edges and vertices. Furthermore, these Platonic polyhedral systems can always ultimately be transformed into at least one of the three possible cases of fundamental omnitriangulated structural systems, viz. the tetrahedron, the octahedron and the icosahedron. Finally, the periodic relations inherent in these rational transformations can be reduced to "Structural Quanta". Next, the dynamic topological frame models complex are constructed by the complementary allspace-fillers of Platonic and Archimedean polyhedral systems, which demonstrate the reciprocal allspace-filling transformations of four dimensional mobility.

INTRODUCTION

In topology, Euler says in effect, all visual experiences can be resolved into three unique and irreducible aspects; vertices, faces and edges--or points, areas and edges.
In terms of Synergetics Topology, they are called respectively joints, windows and struts (Fuller 1975a).

Only the topological analysis of synergetics can account for all the multicongruent --two-, three-, fourfold-- topological aspects (Fuller 1979), by accounting for the primitive six vectors and four vertices of tetrahedron inventories of all Platonic and Archimedean polyhedral systems. Their respective primitive inventories of the topological aspect are always present at all phases of the rational convergence transformation of the Platonic and Archimedean symmetrical structural systems, which is demonstrated in "Periodic Table of Synergetics Topology" and proves the quantum phenomena in terms of "3 Ground States" of Synergetics Topology (Kajikawa 1984, 1985).

NEW MODELS OF SYNERGETICS TOPOLOGY

1. Periodic Table of Synergetics Topology: Structural Quanta

Synergetics Topology' always symmetrical, continuous, complimentarily expanding and contracting, inter-transformings disclose a succession of "local way stations" which constitute a

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periodic hierarchy. These progressive arrivals are recognizable as the family of Platonic or Archimedean polyhedra. All the vertices of the successive forms by the angular closing of immediately adjacent edges of the polyhedra are always diminishingly equidistant from the same centers. The vertices of each of these intertransforming states are always positioned in a progressively expanding or contracting sphere while still maintaining the same distances between the adjacent two vertices.

The flexible joined regular and semi-regular unstable polyhedral systems can be folded into those with fewer regular plane windows, accomplishing the symmetrical collapsing of the same kind of regular windows and ultimately into at least one of the multiple congruent tetrahedra, octahedra or icosahedra which are enclosed with the omnitriangulated windows. The only 3 possible omnitriangulated and omniequiangulated structural systems are defined as "3 Ground States" of Synergetics Topology.

Both the flexible joined regular and semi-regular vector-strut models lie with all the joints (vertices or points) in a containing sphere, the circumsphere. Each and every vector-length between the adjacent two joints are equal. There are only 3 possible types of flexible joints, because all of Platonic and Archimedean polyhedral systems can be sorted out by three, four or five vectors around each of their vertices; valency of the system. All models of polyhedral systems used in this research are made of these three types of flexible joints, windows and struts of the same length (Kajikawa & Sagara 1984b, 1985c).

The following are full explanations of the data given in "Periodic Table of Synergetics Topology".

Column 1) shows the polyhedral systems arranged in order of number of edges, from 1 to 18.

Column 2) contains the tetrahedron, the octahedron and the icosahedron constructed from equilateral triangles. They are the only 3 possible omnitriangulated structures in nature.

Column 3) contains the 15 unstable Plato-Archimedean polyhedral systems.

Column 4) gives the geometric name for each of the polyhedral systems.

Column 5) shows the number of edges for each polyhedral system.

Column 6) shows how all of the unstable regular and semi-regular polyhedral systems can be folded into those with fewer plane windows and ultimately into at least one of "3 Ground States" by accumulating vector-struts of the same number at each of their normal vector-struts. The numbers appearing in this column signify the number of multiple congruent polyhedral systems which will be formed by the continuous contracting process with the axial spinnability. The arrows, x ⟶ y mean that the continued folding of the figure x will result in y.

Column 7) show that when the numbers of vector-edges of each polyhedral system is divided by 6, the result will be one or more of the synergetics first four prime numbers, 1, 2, 3, 5 or multiples thereof. There are either six vectors or none. Six vectors equal one minimum structural system. 6 edge-vectors = 1 structural quantum. (quantum means one of the small subdivisions of a quantized physical magnitude) The definable system of all Plato-Archimedean polyhedra is tetrahedrally coordinate in rational number increments of the tetrahedron.

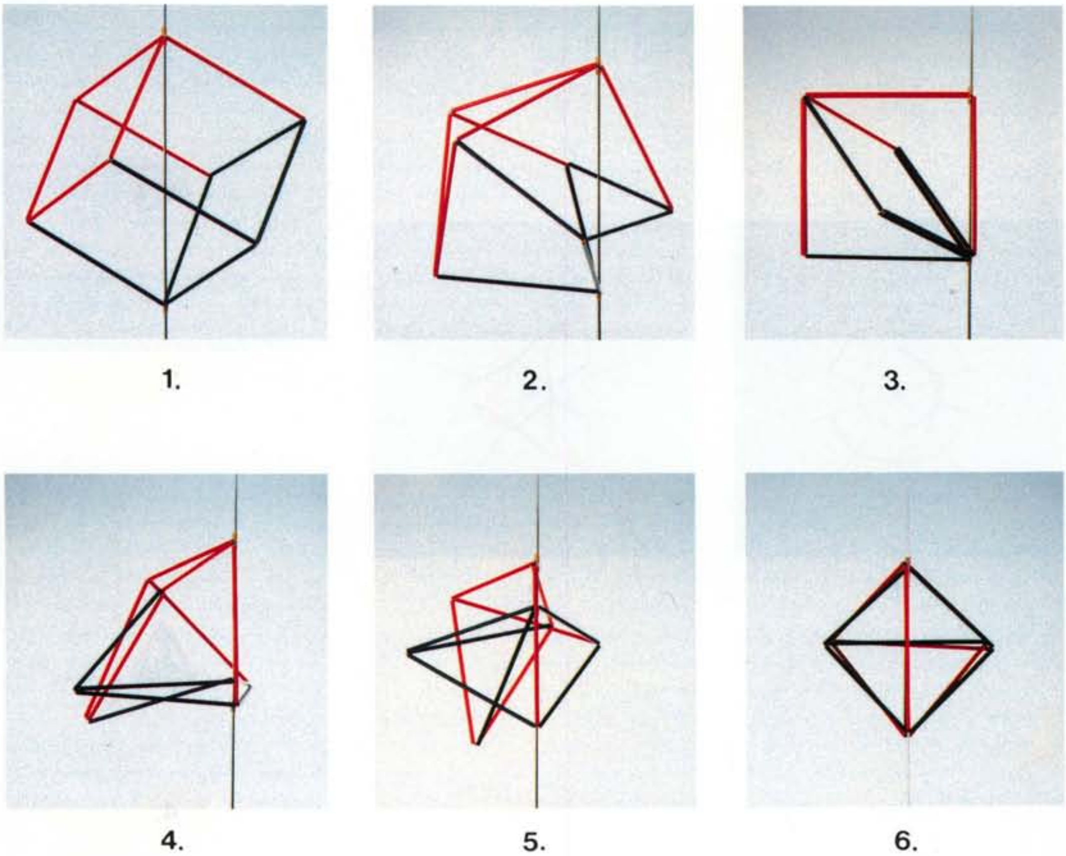
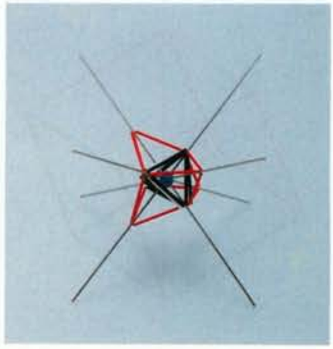


Fig. 1. Symmetrical Contraction of Cube with Axial Spinnability of Two Poles: Tetrahedral Progression

The axis transfixes the cube through the opposing north and south poles (1). If each pole rotates about the axis in the opposite direction by the angular closing of adjacent edges (1), the cube will contract symmetrically to bring together the two opposed vertices which lie on the diagonal of the square (2) until it becomes first the incomplete octahedral phase (3). In this case the two sets of double edges suggest polarization. Next, as the two poles approach each other on the axis to come together the other pairs of opposing vertices (4) (5), the cube folds into two congruent tetrahedra(6). We have arrived at the tetrahedron as a precessional results. The double tetrahedra is the limit case of contraction that expands again symmetrically only to contract once more to become the other double tetrahedra. The cube consists of a positive (red) and a negative (black) tetrahedron and is an indivisible unity. In other words, since the total number of edges and vertices of the cube is exactly twice that of the tetrahedron, we can refer to the cube as "two quanta" in our tetrahedral system. Ranking the models in terms of the number of edges, column 5 of the table shows how 18 types of Platonic and Archimedean polyhedra are all, without exception, composed of edges whose numbers are multiples of six. Multiplication occurs only through the rational fractionation of the complex unity of the minimum prime structural systems.



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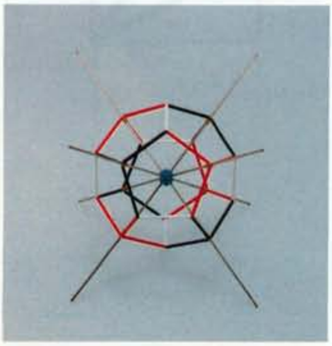

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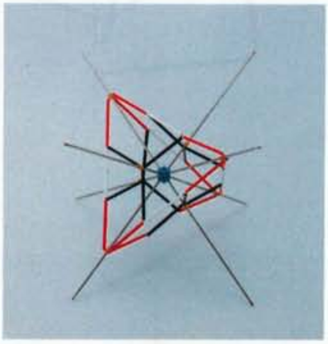


2.

4.

Fig. 2. Four Spoke-Axes of Dodecahedron with Rotating Vertices: Tetrahedral Progression

If four black vertices which have a tetrahedral configuration in the dodecahedron (1) slide inwardly and other four red vertices slide outwardly with spinning on eight spoke-axes forming four axes in right direction, the whole system becomes first the three frequency tetrahedral phase (2). And if four red vertices start to spin inwardly toward "the right" on four axes, the whole system will contract until it becomes the complex unity of the tetrahedra (3). Only by abandoning the four spoke-axes in the system, the dodecahedron will ultimately become the five congruent tetrahedra with spinning on the remaining four spoke-axes (4).

There is the right-handed dodecahedron. To collapse a dodecahedron into five congruent tetrahedra so as to preserve the color symmetry at each of tetraedges (the combination of two red edges, two black edges and one white edge), we must rotate it left or right on the axis produced by treating the two vertices of the dodecahedron as N and S pole respectively.

The significance of "the additive twoness" in Euler's formula V+F=E+2 is also borne out here in the function of the north and south polarity. In the transformation of the dodecahedron, the polarity of the same two opposed vertices is always preserved and finally the two poles overlap on the axis to arrive at the most primitive state. The polarity is inherent in congruence, disclosing the most stabilized symmetrical state of numbered synergetics topological hierarchy.

Column 8) shows that in a quantum leap, the increase or decrease in the number of quanta will always appear in the order of the synergetics first four prime numbers, 1, 2, 3 or 5.

2. Basic Frame Models of Synergetics Topology

We can thread a nylon string through each of the 12 equal-length tubes twice to make a loop and fasten them together with three tubes joined at each of 8 corners to make the cube, which proves to be structurally unstable (See Fig. 1). The tubular frame models of all Platonic and Archimedean polyhedra can be constructed by using this loop-ligature technique (Kajikawa 1983, Kajikawa & Sagara 1984a)

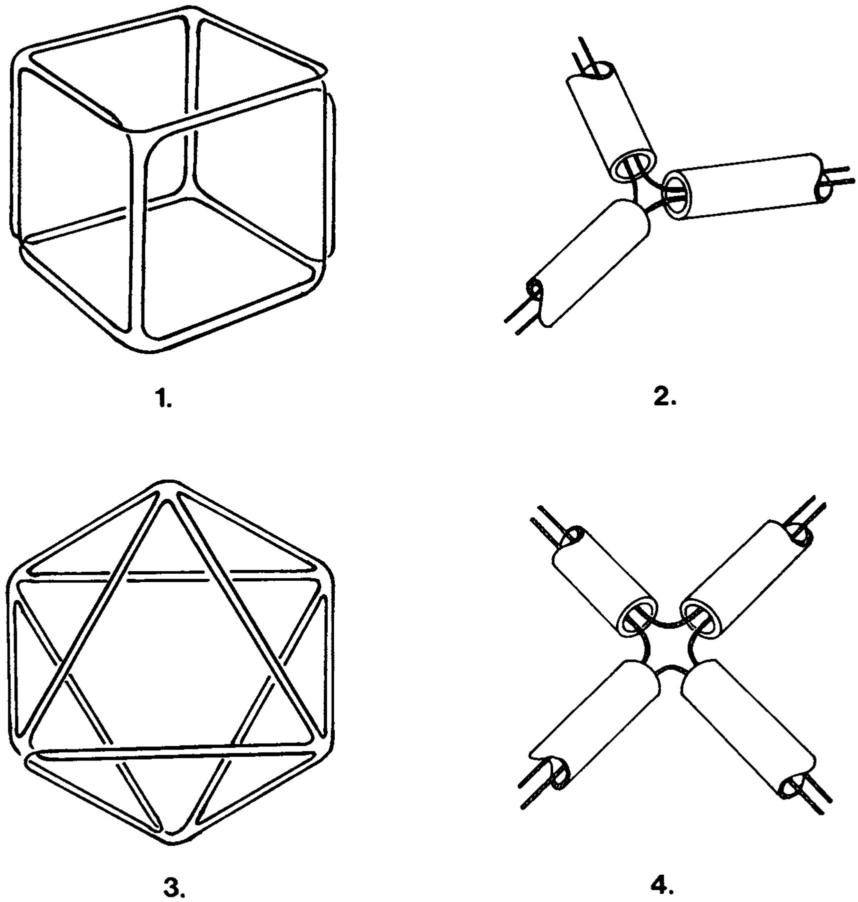
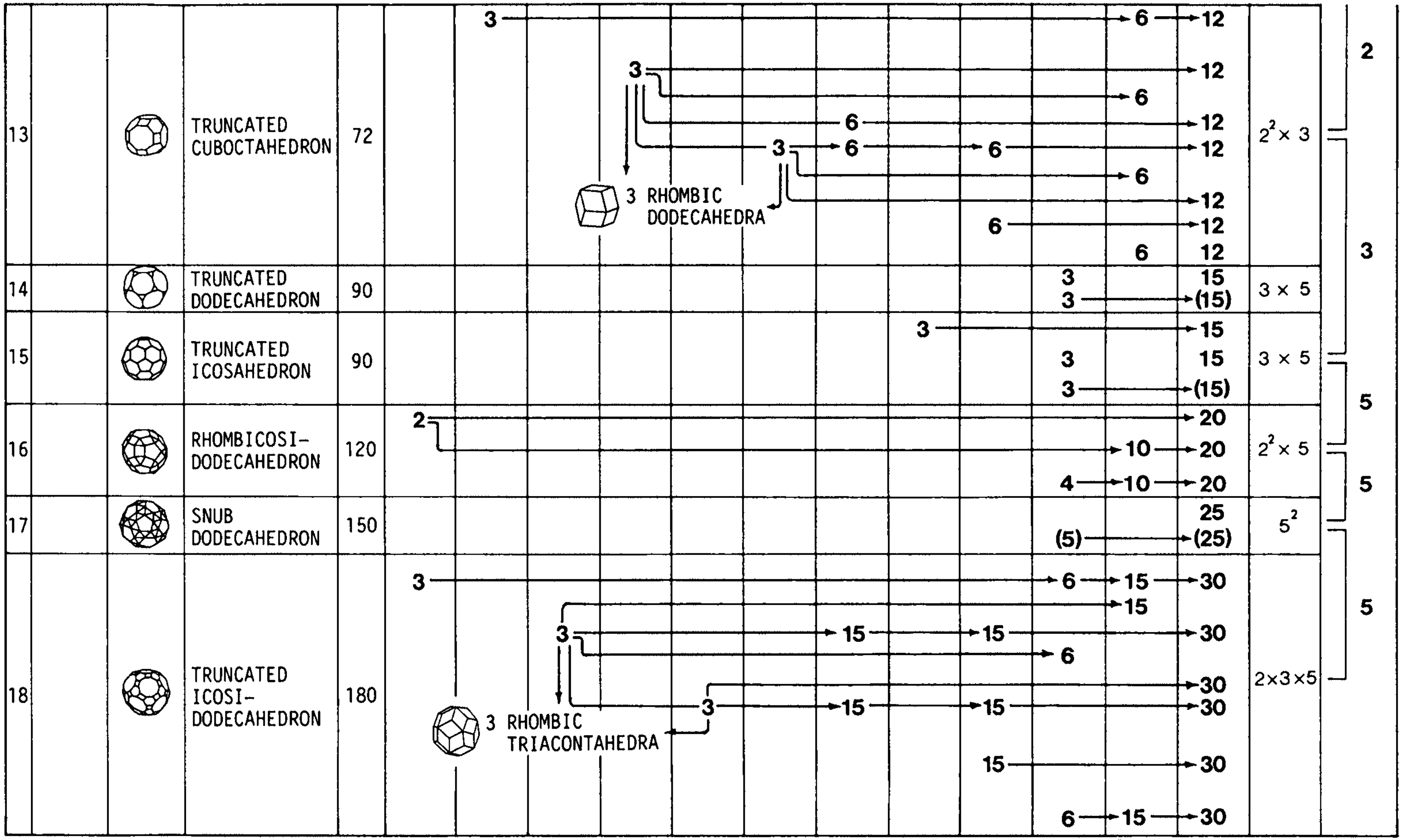


Fig. 3. A Loop Formed by Stringing Twice Through Each Tube

(1) A loop of the cube.
(2) Detail of the loop joint connected with three tubes.
(3) A loop of the octahedron.
(4) Detail of the loop joint connected with four tubes: The string always turns left or right at each corner to make a loop.

Periodic Table of Synergetics Topology: Structural Quanta

1	2	3	4	5	6												7	8
	3V − 6 = E V = Vertices E = Edges	3V − 6 > E	Name of polyhedron	The number of vector-struts	The number of congruent polyhedra: Structural Quanta												Topological synergetics hierarchy	Quantum leap of first four prime numbers
					2 Archimedian polyhedra		5 Doubling Platonic polyhedra					5 Platonic polyhedra						
														3 Ground States				
1			TETRAHEDRON	6												1	1	1
2			OCTAHEDRON	12											1 →	(2)	2	
3			HEXAHEDRON	12									1 →			2	2	1
4			TRUNCATED TETRAHEDRON	18												3	3	
5			CUBOCTAHEDRON	24		1 →									2 →	4 4	2^2	1
6			ICOSAHEDRON	30										1 →		(5)	5	1
7			DODECAHEDRON	30								1 →				5	5	
8			TRUNCATED OCTAHEDRON	36							3 →		3 → 3 →		3 →	6 6 6	2 × 3	1
9			TRUNCATED CUBE	36											3 →	6	2 × 3	2
10			RHOMBI-CUBOCTAHEDRON	48		2 →									4 → 4	8 8 8	2^3	
11			SNUB CUBE	60											5 (5) →	10 (10)	2 × 5	2
12			ICOSI-DODECAHEDRON	60	1 →									(2) →	5 →	(10) (10)	2 × 5	



Each of the 5 Doubling Platonic polyhedra binds two struts at each of the midpoints of its edges. Some of the transformations of Nos. 2, 6, 11, 12 and 17 require the detaching and rejoining of the vertexial connections. They are represented by means of ( ),i.e. irreversibility.

3. Rotation of Four Axes of Truncated Cube: Articulation of Eight Triangular Faces with Twelve Edges

We can have a truncated cube model made out of tubular frames colored in red, black and white with each of the eight transparent plastic triangles connected by four axes with a journal to slide on the shafts. Each shaft consists of a stainless steel rod which is perpendicular to two of the eight triangular faces. This model was invented in 1984-85 (Kajikawa & Sagara 1985b).

The term "truncated" in the names of most of Archimedean polyhedra refers to the new faces created by lopping off the vertices or the edges of the solid. However, in the dynamic frame models the lopping off neither increases nor decreases the total number of faces. By rotating the regular polygons we merely bring about either an expansion or a contraction of the model itself. By means of the rotation we are doing no more than opening or closing windows in the model. In the various contracting phases, each one of their vertices brings about a further circumspherical condition to accommodate the whole motion. The instantaneous appearance of the next neighboring state in its simplest and completely symmetrical condition is what we mean by a "way-station" state with structural quanta.

It is visually evidenced that at the "way-station" states in "Periodic Table of Synergetics Topology", the smoothly omni-intertransforming is four-dimensioned, accommodated by local rotations around four axes of the system. Rotation of four axes of the truncated cube demonstrates the progressive arrivals at these various convergent-divergent transformations.

A. If four red triangles of the model spin in left direction and four black triangles spin in right direction inwardly on four axes, the whole system will contract symmetrically until it becomes first the incomplete rhombicuboctahedral phase and finally the icosahedral phase. At this stage, the icosahedron has six sets of double white edges and twenty-four edges comprising eight sets of red and black triangles with eight spokes forming four axes running through the centers of the uncolored areas of the transparent plastic triangles. There is a direction of spin that throws a twist into the system -- positive and negative. The right-handed icosahedron and the left-handed icosahedron are not the same. We can see that there are really two different icosahedra by means of coloring the vectors to identify them (Kajikawa & Sagara 1985a) (See Fig. 4a).

B. If all the triangles of the truncated cube model spin inwardly on four axes in the same direction, the whole system will also contract symmetrically until it becomes the incomplete right-handed or left-handed snub cube phase, and finally the octahedron phase. At this stage, the vector edges have tripled. There is also a direction of spin that throws a twist into the system --positive and negative. The right-handed octahedron and the left-handed octahedron are not the same (See Fig. 4b).

C. If four red triangles of the truncated cube model spin inwardly in left direction and four black triangles slide inwardly along the four axes without spinning, the whole system will contract symmetrically until it becomes the cuboctahedral phase, which has the four sets of double-edged red and white triangles and the four sets of single-edged black triangles (See Fig. 4c).

D. The eight triangles are always linked with twelve white edges between each of the two adjacent triangles and when one triangle is turned in the opposite direction, the others also rotate automatically to interlock the whole system. This link motion reverses the system to the potential alternate cubocta-

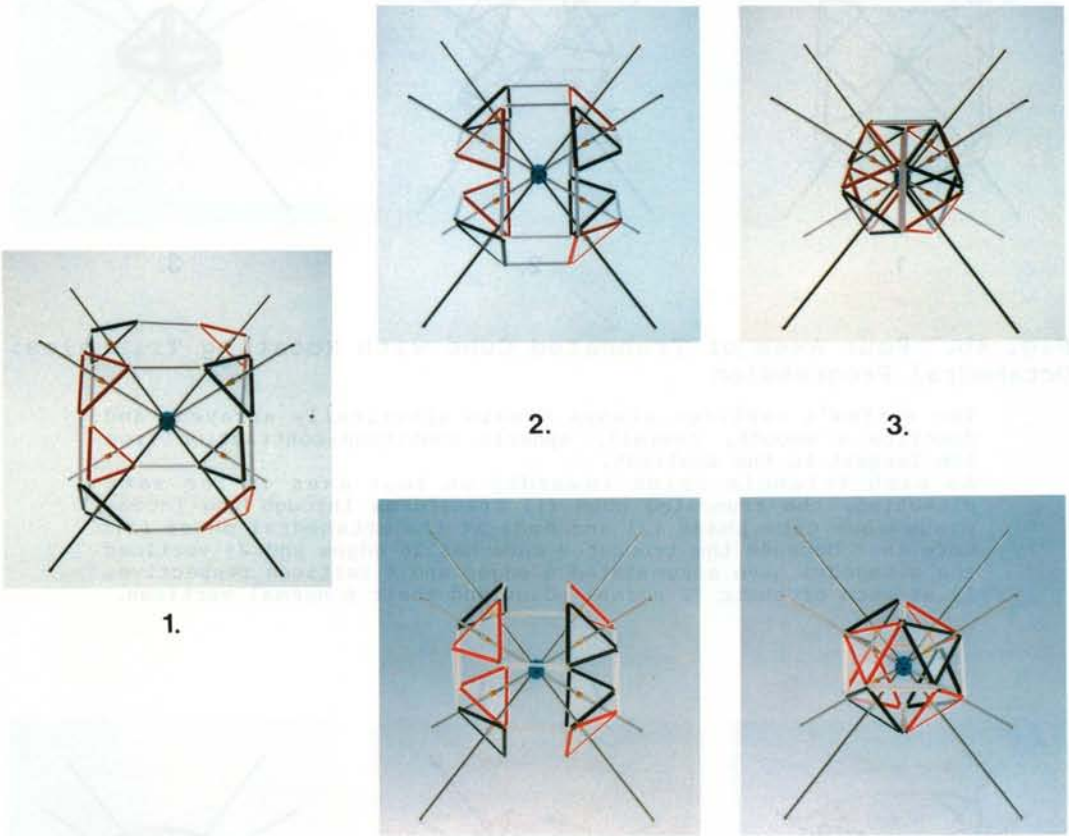
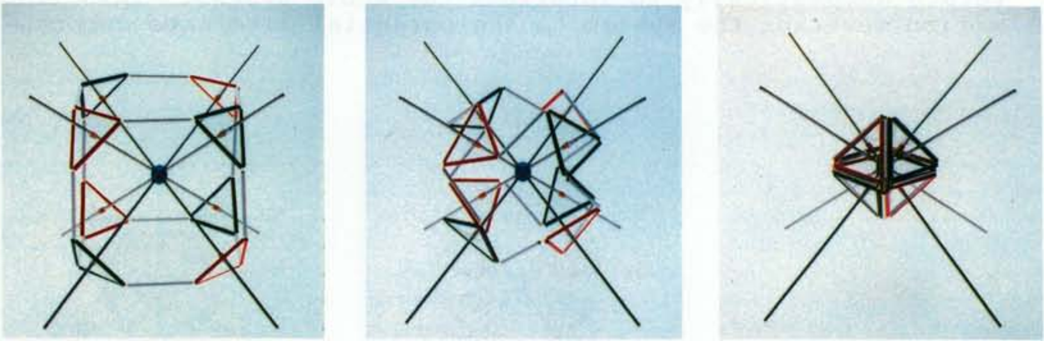


Fig. 4a. Four Axes of Truncated Cube with Rotating Triangles: Icosahedral Progression

If the truncated cube is constructed with 36 circumferential vectors joined at each of 24 corners, each triangle rotates about 8 spokes forming 4 axes, meeting at its center, and approaches its center due to the instability of the octagonal faces. Its various phases are shown in both left- and right-handed contraction.

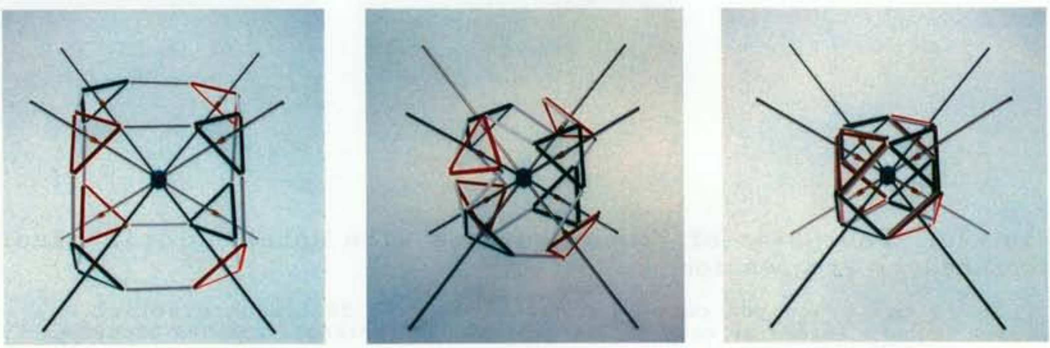
(1) Truncated cube phase: the beginning of the transformation.
(2) The incomplete rhombicuboctahedral phase: When the short diagonal dimension of the octagonal face is equal to the truncated cube's edge length, new 18 square faces are formed.
(3) Icosahedral phase: There are no more octagons and squares. We have a condition of omnitriangulation. Around every vertex we can always count five. Note that in both left- and right-handed case three pairs of opposite doubling white edges which are parallel to one another suggest the alternate skew-transformation by precessional rotation.



1. 2. 3.

Fig. 4b. Four Axes of Truncated Cube with Rotating Triangles: Octahedral Progression

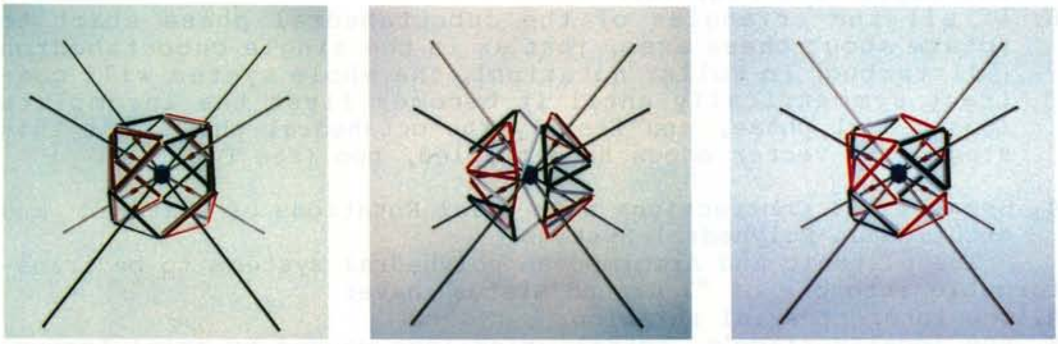
The system's vertices always remain spherically arrayed, and describe a smooth, overall, spheric continuum-contraction from the largest to the smallest.
As each triangle spins inwardly on four axes in the same direction, the truncated cube (1) transforms through the incomplete snub cube phase (2) and ends at the octahedral phase (3). Note that because the truncated cube has 36 edges and 24 vertices the octahedra have accumulated 3 edges and 4 vertices respectively at each of their 12 normal edges and their 6 normal vertices.



1. 2. 3.

Fig. 4c. Four Axes of Truncated Cube with Rotating Triangles: Cuboctahedral Progression

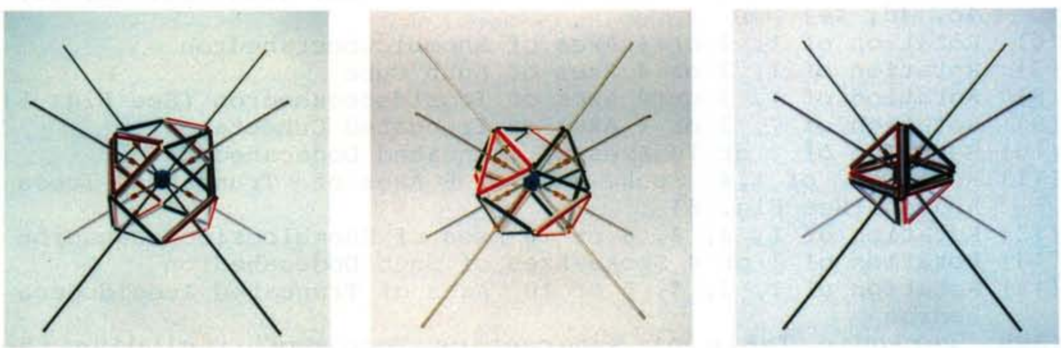
The four red triangles of the truncated cube (1) spin inwardly in right or left direction and the four black triangles slide inwardly along the four axes without spinning (2) until the whole system becomes the cuboctahedral phase (3).



1. 2. 3.

Fig. 4d. Rotation of Triangles on Four Axes: Alternation of Color Symmetry

When one of the red triangles of the cuboctahedral phase (1) is turned in the opposite direction, the other triangles also rotate automatically to interlock the whole system (2). Finally this link motion reverses the system to the alternate cuboctahedral phase again (3).



1. 2. 3.

Fig. 4e. Four Axes of Truncated Cube with Rotating Triangles: Intermediate Transformations as Jitterbug System

Due to the instability of the square faces, there is a further contraction toward the octahedral phase. This symmetrical transformation is similar in the convergent-divergent primitive states which have been referred to as the "jitterbug" (Vector Equilibrium) by R.B.Fuller and the structural quanta in the stabilized states are different.

(1) Vector Equilibrium (cuboctahedron) phase: Note that the only four triangle parts have the doubling of the edges.
(2) Icosahedral phase: When the short diagonal dimension of the quadrilateral face is equal to the vector equilibrium's edge length, twenty equilateral triangular faces are formed.
(3) Octahedral phase: Note the tripling of the edges.

hedral condition again. But the new cuboctahedral phase have completely rearranged the color symmetry of the doubling of the edges (See Fig. 4d).

E. If all the triangles of the cuboctahedral phase start to rotate about these axes, just as in the single cuboctahedron ("Jitterbug" in Fuller notation), the whole system will contract symmetrically until it becomes first the incomplete icosahedral phase, and finally the octahedral phase. At this stage, the vector edges have tripled, too (See Fig. 4e).

4. Symmetrical Contractions with Axial Rotations of Platonic and Archimedean Polyhedral Systems

The platonic and Archimedean polyhedral systems to be transformable into one of "3 Ground States" have:

A. the inherent axial rotation.
B. the local-surface's spiral wrinklings caused by axial torque, when one pole spins right and the other spins left.
C. the polarity that is inherent in congruence.
D. the symmetrical transformation positioned in a sphere that is progressively expanding or contracting.

There are 14 new dynamic frame models of Synergetics Topology.

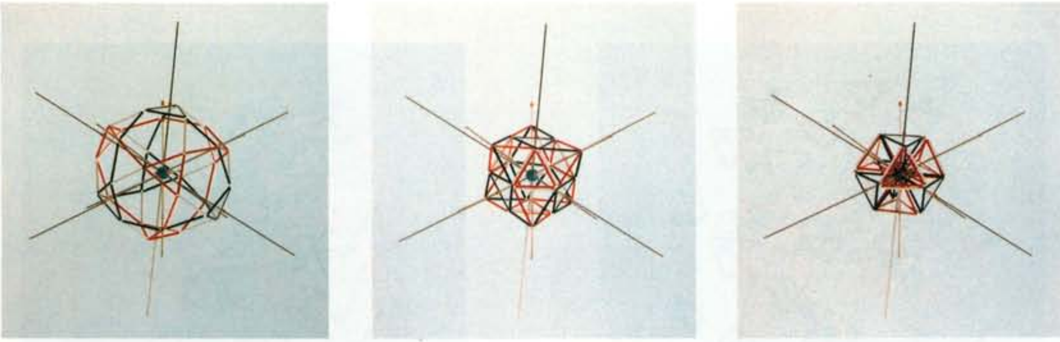
(1) Rotation of 1 Axis of Cube (See Fig. 1)
(2) Rotation of 4 Spoke-Axes of Truncated Tetrahedron (See Fig. 9)
(3) Rotation of 1, 4 Spoke-Axes or 4 Axes of Dodecahedron (See Fig. 2)
(4) Rotation of 1, 3 or 4 Axes of Truncated Octahedron
(5) Rotation of 1 or 4 Axes of Truncated Cube (See Fig. 4a, 4b, 4c, 4d, 4e)
(6) Rotation of 1, 3 or 4 Axes of Rhombicuboctahedron
(7) Rotation of 1, 3 or 4 Axes of Snub Cube
(8) Rotation of 1, 3 or 4 Axes of Icosidodecahedron (See Fig. 5)
(9) Rotation of 1, 3 or 4 Axes of Truncated Cuboctahedron
(10) Rotation of 1 or 10 Axes of Truncated Dodecahedron
(11) Rotation of 1,4 Spoke-Axes or 6 Axes of Truncated Icosahedron (See Fig. 6)
(12) Rotation of 1, 3, 4, 6 or 10 Axes of Rhombicosidodecahedron
(13) Rotation of 1 or 4 Spoke-Axes of Snub Dodecahedron
(14) Rotation of 1, 3, 4, 6 or 10 Axes of Truncated Icosidodecahedron

(See "Periodic Table of Synergetics Topology" including the axial spinnability of vertices or faces.)

RECIPROCAL ALLSPACE-FILLING TRANSFORMATIONS

1. Allspace-filling Transformations of Truncated Cube

Because the truncated cube and the octahedron will fill space, it is possible to visualize a device for the continuous allspace-filling "truncated cube" transformations. If we join many truncated cubes at their regular octagonal faces in a double allspace-filling arrangement, the triangular faces form octahedral voids. After we put together a large omnidirectional complex of sets of four axes and eight transparent plastic triangles with twelve edges, we can interconnect the triangles of the octahedral voids from set to set with alternate sets of twelve edges (See Fig. 7-1). As the truncated cubes contract towards each center, just as in the "single truncated cube", they transform through the rhombicuboctahedral phase and the cuboctahedral



1. 2. 3.

Fig. 5. Four Axes and Three Axes of Icosidodecahedron with Rotating Triangles and Vertices

It is possible to arrange 20 triangles around pentagons. The shape is the icosidodecahedron. This model is constructed with eight spokes forming four axes and six spokes forming three axes, meeting at its center, which pass through the centers of eight triangles and six vertices respectively (1).

If six vertices which have a octahedral configuration in the icosidodecahedron spin outwardly on the three axes and eight triangles spin inwardly on four axes in the opposite direction respectively, the whole system will become first two frequency right- or left-handed octahedral phase (2). And if six vertices start to approach its center, the whole system will contract to become the cuboctahedral phase (3).

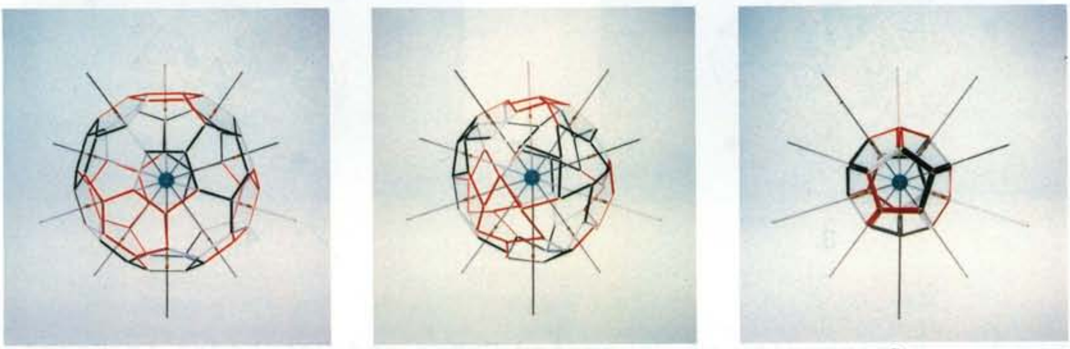


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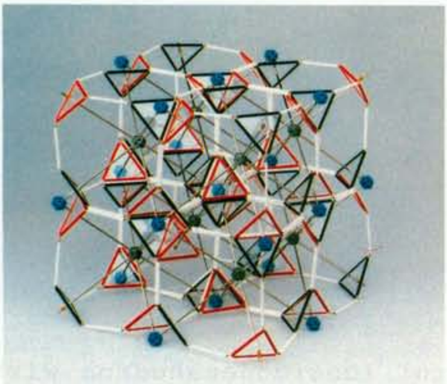
Fig. 6. Six Axes of Truncated Icosahedron with Rotating Pentagons: Dodecahedral Progression

The shape is the truncated icosahedron (1). When a model is constructed with 12 spokes ,i.e. six axes, meeting at its center, which pass through the centers of each pentagon, a symmetrical behavior results. If the twelve pentagons of the model spin inwardly on six axes in the same direction, the whole system will contract symmetrically until it becomes first the incomplete right-handed or left-handed snub dodecahedral phase (2), and finally the dodecahedral phase (3). Note that because the truncated icosahedron has 90 edges and 60 vertices the dodecahedra have accumulated 3 edges and 3 vertices respectively at each of their 30 normal edges and 20 normal vertices. 3 congruent dodecahedra can be ultimately transformed into 15 congruent tetrahedra in similar progressive way of a normal dodecahedron by replacing the six axes with the four spoke-axes (See Fig. 2).



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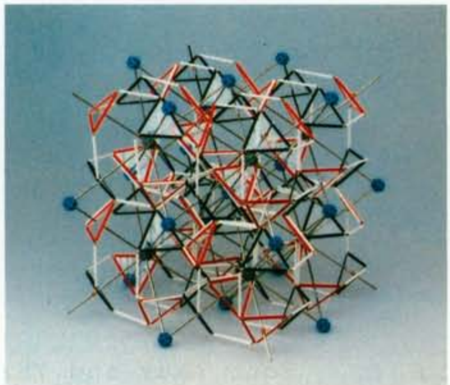

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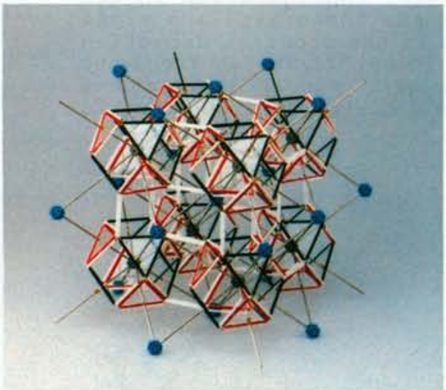

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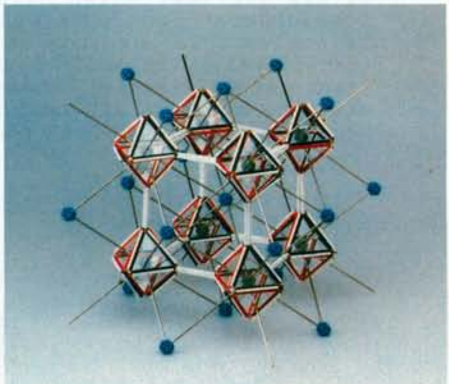
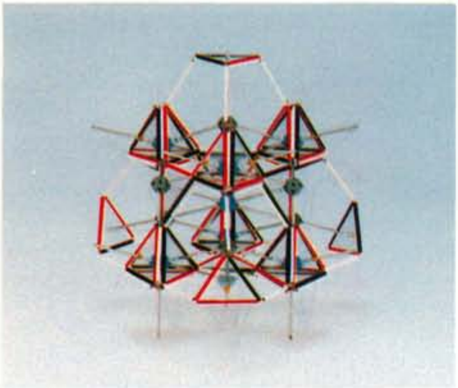


4.

Fig.7. Reciprocity of Truncated Cubes and Octahedra in Space-Filling "Truncated Cube"

In the allspace-filling "truncated cube" transformation, we find that if one force is applied to one triangle of one open truncated cube, the actual truncated cube closes to become an octahedron throughout the whole system and that each one of their vertices brings about a further spherical condition to accommodate the whole motion with a large omnidirectional complex of sets of four axes. The original truncated cubes (1) contract through rhombicuboctahedral phase (2) and cuboctahedral phase (3) and ultimately become octahedra (4). There is a complete change of the two figures. There is also a force distribution lag in the system. The distance between each of the two adjacent cores of the models is always constant in both double (1) (4) and triple (2) (3) allspace-filling arrangements.



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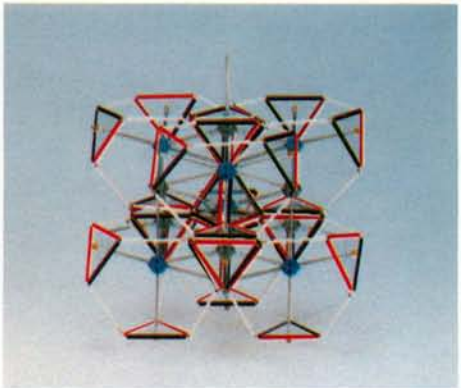
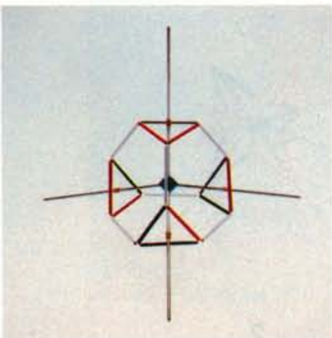


2.

Fig. 8. Reciprocity of Truncated Tetrahedra and Tetrahedra in Space-Filling "Truncated Tetrahedron"

Because the truncated tetrahedron and the tetrahedron will fill space, it is possible to visualize a space-filling "truncated tetrahedron" transformation. If we combine truncated tetrahedra on their regular hexagonal faces in a dual allspace-filling arrangement, the triangular faces form tetrahedral voids (1). As the truncated tetrahedra contract towards each center, just as in the single "truncated tetrahedron"(See Fig.9), they transform through the icosahedral phase , and end at the tetrahedral phase (2). Every truncated tetrahedron will become a tetrahedron and every tetrahedron will become a truncated tetrahedron on a large omnidirectional complex of the sets of four spoke-axes, because in the allspace-filling transformation there are constant numbers of actual triangular faces. There is a complete change of the two figures and a force distribution lag in the system.



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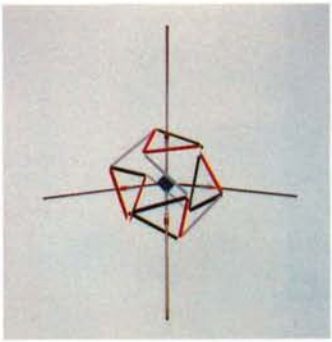

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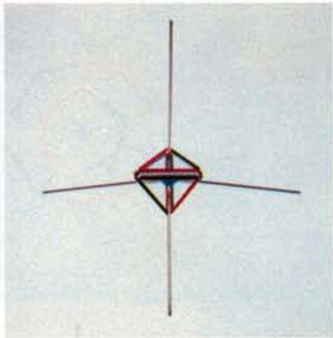
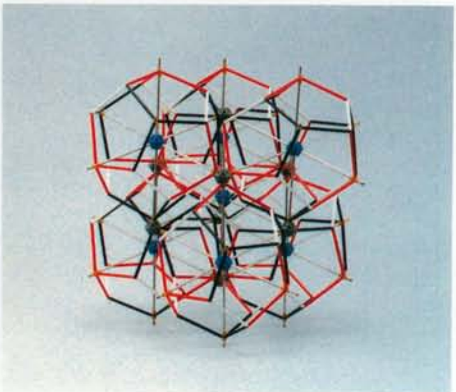


3.

Fig. 9. Four Spoke-Axes of Truncated Tetrahedron With Rotating Triangles

If four triangles of the the truncated tetrahedron (1) spin in the same direction, the system will contract symmetrically until it becomes the incomplete icosahedral phase (2) and finally the tetrahedral phase (3). At this stage, the vector edges have tripled.



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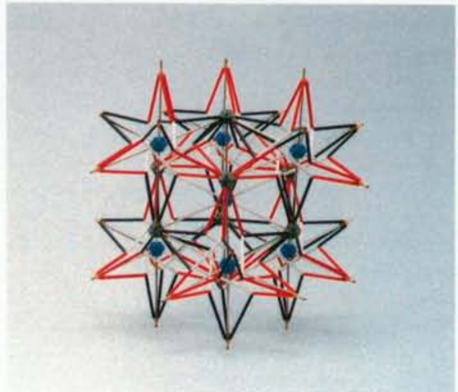
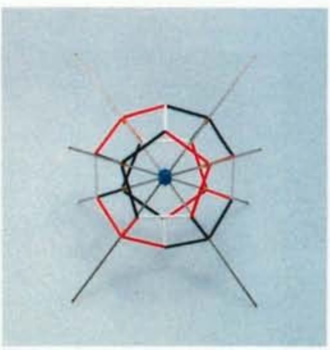


2.

Fig. 10. Transformation of Dodecahedron and Great Stellated Dodecahedron as Space-Filling "Cubic Disequilibrium"

If we join many dodecahedra with four axes at the parts of their regular pentagonal faces in a dual allspace-filling arrangement, the pentagonal faces form the incomplete great stellated dodecahedral voids with four axes (1).
In the allspace-filling "cubic disequilibrium" transformation, the dodecahedra contract to become the incomplete great stellated dodecahedra, and the original incomplete great stellated dodecahedra expand and ultimately become dodecahedra (2). There is a rotationlessly complete change of the two figures.



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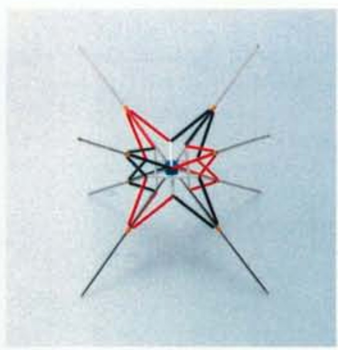

2.

Fig. 11. Reciprocity of Cubic Disequilibrium with Icosahedral Order

If the dodecahedron (1) collapses toward its center to preserve the three parallel pairs of white edges, it becomes the incomplete great stellated dodecahedron (2). At this stage, both ends of six white edges form twelve vertices of the icosahedral phase.

phase (See Fig. 7-2, 7-3), which appears throughout the whole system as a triple allspace-filling arrangement of cuboctahedra, rhombicuboctahedra and cubes, and end at the octahedron phase (See Fig. 7-4), which appears again as a double allspace-filling arrangement of truncated cubes and octahedra .

In other words, the original octahedra expand and ultimately become truncated cubes. Every truncated cube will become an octahedron and every octahedron will become a truncated cube on a large omnidirectional complex of sets of four axes. There is a complete change of the two figures. This oscillating motion makes an expanding and contracting system. In doing so, with a oscillating system and a pulsating circumspherical expansion-contraction going on everywhere locally, the system becomes an optically pulsating circumsphere.

Each exterior octahedron is a contracted truncated cube and is approximately one of the spaces between the circumspheres of the actual truncated cubes which overlap locally. Each octahedron thus becomes available as a potential alternate new circumsphere when the old circumspheres become spaces.

2. Other Types of Reciprocal Allspace-Filling Transformations

"Rotation of 4 Axes of Vector Equilibrium" (vector equilibrium means cuboctahedron) was discovered by R.B.Fuller in 1944 (Fuller 1975b). In 1976 he further conceived and designed a limited edition of the metal sculpture "Complex of Jitterbugs", demonstrating 4-D wave generation as the reciprocity of cuboctahedron and octahedra in a allspace-filling jitterbug. This is the first complex model of synergetics topology with four axes, which has been provided with omnidirectional conceptual comprehension of the separate and combining transformations of local energy events (Fuller 1975c).

By using the other types of the reciprocal allspace-filling transformations, which I had the good fortune to discover and develop in 1984-85 (kajikawa & Sagara 1985), we make a single energy action in the spinning system and a complete omnidirectional rotation occurs so that every one of the faces of the complementary allspace filler can shuttle back and forth. Synergetics Topology adds six more reciprocal space-filling transformations:

(1) truncated tetrahedron + tetrahedron (See Fig. 8)
(2) truncated cube + octahedron (See Fig. 7)
(3) truncated octahedron + cube + truncated cuboctahedron
(4) cube + rhombicuboctahedron + tetrahedron
(5) truncated octahedron + cuboctahedron + truncated tetrahedron
(6) rhombicuboctahedron + cuboctahedron + cube

The distance between each of the two adjacent cores is always constant in both double and triple allspace-filling arrangements.

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CREDIT FOR THE CHART ILLUSTRATIONS

Popko, E. (1968): Basic Polyhedra. Geodesics. [Detroit. University of Detroit Press. 106] : Fig. No. 1

New Models of Synergetics Topology
and
Their Reciprocal Allspace-Filling Transformation

NEW MODELS OF SYNERGETICS TOPOLOGY

翻訳要旨

シナジェティクストポロジーモデルと、空間充填におけるその相互変換

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プラトン、アルキメデスの多面体において、長さの等しい辺を 3,4 または 5 本ずつその端部において相互に結合して角度的に自由に変化する柔軟な頂点によって構成すると、正四面体、正八面体、正二十面体を除くすべての構造的に不安定な多面体の形態は、変形させることができる。その時、頂点が半径の収縮する同一の球面上に常に位置し、隣合う２頂点間を等距離に維持しながら、回転軸を対称的にスピンして、多面体の面数を減少させるトポロジーの存在が発見された。1944 年に R.B.Fuller が、アルキメデスの準正多面体の１つである立方八面体を、対称的に二重の正八面体に変換可能なことを示した発見と、これらの発見を結合し一般化したものが、シナジェティクストポロジーである。即ち、正および準正多面体はすべて、正四面体、正八面体、正二十面体のいずれか１つに連続的に変換される。これらの力学的に安定した３つの基底状態が、すべて 6 の整数倍から成り立つ辺数をもつことによる辺における序列化によって、多面体相互間の周期律が出現する。それは、シンメトリーの階層構造を再現している。

さらにこれらの回転軸をもつ相補的なプラトン、アルキメデスの多面体の動力学的なフレームモデルを、その中心核間距離が一定となるように、周期的に空間に配置し、相互に連結することによって、回転軸上のすべての面が回転し連動して、一方が収縮する時他方が拡大して形態が入れ換わる、新たな動力学的な空間充填システム群の存在が明らかになった。このような空間充填システムの相互変換の過程には、プラトン・アルキメデスの多面体の空間充填におけるすべてのパターンが潜んでいる。